**Abstracts**

**Speaker:** Andrew Sutherland, MIT.

**Title:** Isogeny volcanoes.

**Abstract:** The remarkable structure and computationally explicit form of isogeny graphs of elliptic curves over a finite field has made them an important tool for number theorists and cryptographers alike. I will give an overview of the theory behind these graphs and then examine several recent applications where substantial (in some cases dramatic) computational improvements have been achieved by exploiting this theory.

**Speaker:** David Zureick-Brown, Emory University.

**Title:** 2-adic images of Galois representations.

**Abstract:** We give a classification of all possible 2-adic images of Galois representations associated to elliptic curves over $\mathbb{Q}$. To this end, we compute the ‘relevant’ tower of 2-power level modular curves, develop new techniques to compute their equations, and classify rational points on these curves. This is joint work with Jeremy Rouse.

**Speaker:** Michael Griffin, Emory University.

**Title:** Weierstrass mock modular forms and elliptic curves.

**Abstract:** Mock modular forms, which give the theoretical framework for Ramanujan’s enigmatic mock theta functions, play many roles in mathematics. We study their role in the context of modular parameterizations of elliptic curves $E/\mathbb{Q}$. We show that mock modular forms which arise from Weierstrass-functions encode the central $L$-values and $L$-derivatives which occur in the Birch and Swinnerton-Dyer Conjecture. By defining a theta lift using a theta kernel recently studied by Hoevel, we obtain canonical weight 1/2 harmonic Maass forms whose Fourier coefficients encode the vanishing of these values for the quadratic twists of $E$. We employ results of Bruinier and Ono, which builds on seminal work of Gross, Kohnen, Shimura, Waldspurger, and Zagier. For $E$ with CM, we also obtain $p$-adic formulas for the corresponding weight 2 newform using the action of the Hecke algebra on the Weierstrass mock modular form. This is joint work with Claudia Alfes, Ken Ono, and Larry Rolen.
**Speaker:** Sarah Trebat-Leder, Emory University.

**Title:** Elliptic curves with full 2-torsion and maximal adelic Galois representations.

**Abstract:** In 1972, Serre showed that the adelic Galois representation associated to a non-CM elliptic curve over a number field has open image in $GL_2(\hat{\mathbb{Z}})$. In his thesis, Greicius develops necessary and sufficient criteria for determining when this representation is actually surjective and exhibited such an example. However, verifying these criteria turns out to be difficult in practice; Greicius described tests for them that apply only to semistable elliptic curves over a specific class of cubic number fields. We extend Greicius’ methods in several directions. First, we consider the analogous problem for elliptic curves with full 2-torsion. We obtain necessary and sufficient conditions for the associated adelic representation to be maximal and also develop a battery of computationally effective tests that can be used to verify these conditions. Using these tests, we are able to construct an infinite family of curves over $\mathbb{Q}(\alpha)$ with maximal image, where $\alpha$ is the real root of $x^3 + x + 1$. We also extend Greicius’ tests to more general settings, such as non-semistable elliptic curves over arbitrary cubic number fields.

**Speaker:** John Webb, Wake Forest University.

**Title:** Some new fun facts about eta-quotients.

**Abstract:** Let $\eta(z)$ be Dedekind’s eta-function. For an integer $N \geq 1$, a function of the form $f(z) = \prod_{d|N} \eta(dz)^{r_d}$ with each $r_d \in \mathbb{Z}$ is called an eta-quotient; these functions have proved invaluable in the study of modular forms. If an eta-quotient is a holomorphic weight $k$ modular form on $\Gamma_0(N)$, we find a bound for $\sum_{d|n} |r_d|$. We also show that any modular form in the space $M_k(\Gamma_0(N))$ with integral Fourier coefficients which is non-zero on the upper-half plane is an eta-quotient, generalizing a result of Kohnen. As an application, we calculate the cuspidal $\mathbb{Q}$-rational torsion subgroup of $J(\Gamma_0(2^n))$ for all $n \geq 1$. This is joint work with Jeremy Rouse.

**Speaker:** Jesse Thorner, Emory University.

**Title:** The explicit Sato-Tate conjecture.

**Abstract:** Let $f(z) = \sum_{n \geq 1} a(n)e^{2\pi inz} \in S^\text{new}_k(\Gamma_0(N))$ be a normalized Hecke eigenform without complex multiplication. For a prime $p$, define $\theta_p \in [0, \pi]$ to be the angle for which $a(p) = 2p^{(k-1)/2}\cos(\theta_p)$. The recently-proven Sato-Tate Conjecture states that the sequence $\{\theta_p\}$ is uniformly distributed in the interval $[0, \pi]$ with respect to the Sato-Tate measure $d\mu_{ST} = \frac{2}{\pi} \sin^2(\theta) \, d\theta$. In other words, if $I = [\alpha, \beta] \subset [0, \pi]$, then $\#\{p \in [x, 2x] : \theta_p \in I\} \sim \mu_{ST}(I)(\alpha(2x) - \beta(x))$. Provided that $N$ is squarefree and the symmetric power $L$-functions of $f$ are automorphic and satisfy the Generalized Riemann Hypothesis, we prove a completely explicit error term of the form

$$\left| \#\{p \in [x, 2x] : \theta_p \in I\} - \mu_{ST}(I)(\alpha(2x) - \beta(x)) \right| = O\left( \frac{x^{3/4} \log(Nkx)}{\log(x)^{3/2}} \right).$$

We will discuss an application to the study of the number of representations of a positive integer $n$ by positive-definite, integer-valued quadratic forms.
Speaker: Daniel Fiorilli, University of Michigan.

Title: A probabilistic study of the explicit formula.

Abstract: Chebyshev observed that in initial intervals of the integers, there seems to be more primes of the form $4n + 3$ than of the form $4n + 1$. As it turns out, Rubinstein and Sarnak showed under technical hypotheses that this assertion is true about 99.59% of the time (on a logarithmic scale). Since Chebyshev's observation, many other types of 'arithmetical biases' have been found. As was observed by Mazur and explained by Sarnak, such a bias appears in the count of points on reductions of a fixed elliptic curve and is created by its analytic rank, which is conjecturally equal to its algebraic rank. Interestingly, one can use the language of probability to study such biases, and explain both quantitative and qualitative features. In this talk we will focus on extreme biases (such as the original one found by Chebyshev), which highlight extreme arithmetical contexts such as elliptic curves of high rank or moduli having an exceptional number of prime factors. Time permitting, we will discuss various applications of probabilistic techniques to classical questions of analytic number theory such as the variance primes in arithmetic progressions and the vanishing of $L$-functions at the central point.

Speaker: Caroline Turnage-Butterbaugh, University of Mississippi.

Title: Moments of products of $L$-functions.

Abstract: The Riemann zeta-function $\zeta(s)$ and its generalizations, called $L$-functions, are ubiquitous yet mysterious functions in number theory. These functions can be defined in association with a plethora of mathematical objects, including Dirichlet characters, number fields, and elliptic curves. We consider arbitrary products of $L$-functions attached to irreducible cuspidal automorphic representations of $GL(m)$ over $\mathbb{Q}$. The Langlands program suggests essentially all $L$-functions are of this form. Assuming some standard conjectures, I will discuss how to estimate the continuous moment of an arbitrary product of primitive $L$-functions. This is a generalization of a result of K. Soundararajan and is inspired by the work of V. Chandee.

Speaker: Samuel Gross, Bloomsburg University.

Title: 4959866989151226098104244512918.

Abstract: Let $f(x)$ be a polynomial with non-negative integer coefficients for which $f(10)$ is a prime. A result of A. Cohn implies that if the coefficients of $f(x)$ are $\leq 9$, then $f(x)$ is irreducible. In 1988, M. Filaseta showed that the bound 9 can be replaced by $10^{30}$. Can we do better?
Speaker: Scott Dunn, University of South Carolina.

Title: 8925840.

Abstract: Let \( f(x) \) be a polynomial with non-negative integer coefficients such that \( f(b) \) is prime for some integer \( b \geq 2 \). A. Cohn’s criteria states that if \( b = 10 \) and each coefficient is \( \leq 9 \), then \( f(x) \) is irreducible. This bound has been subsequently improved, in particular in 2012 for \( b = 10 \). M. Cole, S. Dunn, and M. Filaseta in 2013 examined the case of \( b \neq 10 \) and established certain irreducibility criteria, which are the focus of this talk.

Speaker: Joshua Harrington, University of South Carolina.

Title: The reducibility of constant-perturbed products of cyclotomic polynomials.

Abstract: In 1906, Schur raised the question of the irreducibility over \( \mathbb{Q} \) of polynomials of the form \( f(x) = (x-a_1)(x-a_2)\cdots(x-a_n)+1 \), where the \( a_j \) are distinct integers. In this talk, we investigate the analogous question when replacing the linear polynomials with cyclotomic polynomials and allowing the constant perturbation of the product to be any integer \( d \notin \{-1,0\} \).

Speaker: Chad Awtrey, Elon University.

Title: Degree 14 2-adic fields.

Abstract: Fix a prime number \( p \) and a positive integer \( n \). A foundational result in algebraic number theory states that there are only finitely many nonisomorphic extensions of the \( p \)-adic numbers of degree \( n \). Many researchers have focused on developing methods for computing data about these extensions (such as Galois groups and ramification information). In this talk, we’ll survey some of the past research related to computational \( p \)-adic field theory. We will also discuss recent and current work (joint with undergraduates at the speaker’s institution) that has completely classified extensions up through and including \( n = 15 \) along with partial results for \( n = 16 \). As a concrete example, we illustrate several of the tools we have employed to compute Galois groups of degree 14 extensions of the 2-adic numbers.

Speaker: Peter Fletcher, Virginia Tech.

Title: 71023.

Abstract: Let \( p \) be an odd prime. A divisor is a congruence class in the multiplicative group of units of \( \mathbb{Z}/(p) \) that contains a divisor of \( p - 1 \). This math-club level talk examines primes like 71023 that have a divisor \( d \) whose complement is \( d + 1 \). We also consider primes like 112643 that have exactly one divisor that is a generator.
**Speaker:** William Banks, University of Missouri.

**Title:** On repeated values of the Riemann zeta function on the critical line.

**Abstract:** Let $\zeta(s)$ be the Riemann zeta function. In this talk, I will describe some recent joint work pertaining to repeated values of $\zeta(s)$ on the critical line, and I describe our evidence to support the conjecture that for every nonzero complex number $z$, the equation $\zeta(1/2 + it) = z$ has at most two real solutions $t$.

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**Speaker:** Lola Thompson, Oberlin College.

**Title:** Variations on a theorem of Davenport concerning abundant numbers.

**Abstract:** Let $\sigma(n)$ denote the usual sum-of-divisors function. In 1933, Davenport showed that $n/\sigma(n)$ possesses a continuous distribution function. We study the behavior of the sums $\sum_{n \leq x, n/\sigma(n) \leq u} f(n)$ for certain complex-valued multiplicative functions $f$. Our results cover many of the more frequently encountered functions, including $\varphi(n)$, $\tau(n)$, and $\mu(n)$. They also apply to the representation function for sums of two squares, yielding an analogue of Davenport’s result.

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**Speaker:** Thomas Wright, Wofford College.

**Title:** Are there infinitely many elliptic Carmichael numbers?

**Abstract:** Yes.

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**Speaker:** Michael Kelly, University of Texas at Austin.

**Title:** Uniform dilations in high dimensions.

**Abstract:** It is a theorem of S. Glasner that given an infinite subset $X$ of the torus $\mathbb{R}/\mathbb{Z}$ and an $\epsilon$ greater than 0 there exists a positive integer $n$ such that any interval of length $\epsilon$ in $\mathbb{R}/\mathbb{Z}$ contains a point of the set $nX$ (that is, $nX$ is $\epsilon$-dense in $\mathbb{R}/\mathbb{Z}$). The set $nX$ is called a dilation of $X$ by $n$. We will discuss various developments on this topic and I’ll present joint work with Le Thai Hoang where we consider this phenomenon in higher dimensions.

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**Speaker:** Michael Knapp, Loyola University Maryland.

**Title:** 2-adic zeros of additive forms.

**Abstract:** In this talk, we determine the minimum number of variables needed to guarantee that a homogeneous polynomial of the form $a_1x_1^d + a_2x_2^d + \cdots + a_sx_s^d$, with integer coefficients, has a nontrivial 2-adic zero.
**Speaker:** Bobby Grizzard, University of Texas at Austin.

**Title:** Relative Bogomolov extensions.

**Abstract:** A subfield $K \subseteq \mathbb{Q}$ is said to satisfy the Bogomolov property if the absolute logarithmic height of non-torsion points of $K^\times$ is bounded away from 0. This can be generalized to relative extensions by defining an extension $L/K$ to be Bogomolov if the height of points of $L^\times \setminus K^\times$ is bounded away from zero. We’ll introduce a criterion for this property and use it to produce many examples. Our main result is that if there are primes with finite ramification index in $K$, then $K$ admits Bogomolov extensions.

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**Speaker:** Rick Farr, University of North Carolina, Greensboro.

**Title:** Zeros of the derivatives of the Riemann zeta function in the complex left half plane.

**Abstract:** We present zeros of the derivatives of the Riemann zeta function in the left half of the complex plane. Our computations show an interesting behavior of the zeros, namely they seem to lie on curves which are extensions of certain chains of zeros that were observed on the right half plane.

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**Speaker:** Braedon Suminski, Washington & Lee University.

**Title:** Polygonal Sierpiński numbers.

**Abstract:** The number 78557 has the interesting property that for any positive integer $n$, $78557 \cdot 2^n + 1$ is composite. Numbers with this property are called Sierpiński numbers in honor of W. Sierpiński’s work. In this talk, we discuss the existence of Sierpiński numbers in the sequences of triangular numbers, square numbers, pentagonal numbers, and other polygonal numbers.