Animating Class with Computer Graphics

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Why learn math?
WE ALL USE MATH EVERY DAY

to forecast weather

to handle money

we also use math to analyze crime

to reveal patterns

to predict behavior...

FRIDAYS 10/9c

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NUMB3RS

FRIDAYS 10/9c

Just for fun

- Let’s bring some Hogwarts to the classroom.
- Not a Potter fan? It can be just as “magical” in other contexts.
Just for fun

• Let’s bring some Hogwarts into our classes.
• Not a Potter fan? We’ll see such “magic” in other contexts, too.

Picture credit: http://whiskersandpawsshelter.webs.com/apps/photos/photo? photoid=96188780
When life is linear

• Keep in mind all our spells will be cast using only linear algebra!
• How magical, even for Muggles, can linear algebra really be?

Muggle Magic

Picture credit: http://images.wikia.com/harrypotter/images/1/16/Harrypotterwand.jpeg
Where’s the matrix?

- An image results from 3 matrices, $R$, $G$, and $B$ giving the intensity of red, green and blue for each pixel in the image.
- A 400 by 600 pixel image will have three 400 x 600 matrices.

Matrix Addition

- Let’s have Harry disappear behind his invisibility cloak.
- We simply use:
  \[ C = B + F, \]
  where \( C \) is an image to visualize.

Photoshop magic

To form $B$:

![Photo of boy 1] + ![Photo of boy 2] = ![Final photo of boy]

*Picture credit: wired.com and http://www.bbc.co.uk/blogs/ni/Harry-Potter.jpg*
Vexing Combination

We will let $B$ be the first image and $F$ be the final image.
Getting Loopy

Visualize C where

\[ C = B + F \]

between 0 and 1  equals 1 -  

\( B \)  \( F \)
Visualize C where

\[ C = B + F \]

between 0 and 1

equals 1 - C
and gone...

Visualize $C$ where

$$C = 0 \cdot B + 1 \cdot F$$

between 0 and 1

equals $1 - C$
Going for a loop

Visualize C where

\[ C = 1 \cdot B + 0 \cdot F \]
Going for a loop

Visualize C where

\[ C = 0.75 \times B + 0.25 \times F \]
Going for a loop

Visualize C where

\[ C = 0.5B + 0.5F \]
Going for a loop

Visualize $C$ where

$$C = 0.25 \, B + 0.75 \, F$$
Going for a loop

Visualize $C$ where

$$C = 0 \cdot B + 1 \cdot F$$
Vexing Combination

Note, we are using the convex combination:

\[ C = (1-\alpha)B + \alpha F, \]

for \( 0 \leq \alpha \leq 1 \) where \( C \) is an image to visualize.
Teleport

This same idea can enable Harry to travel from one location to another.

Picture credit: http://foreveryoungadult.com/wp-content/upload/2010/11/hogwarts03b.jpg
Desaturate

Start with an image and end with a grayscale version of the same image and you’ll see the color sucked right out of it!
Flipping over Harry

- Imagine at Hogwarts we are learning the levicorpus spell, which flips a person.
- Malfoy intentionally misdirects his spell at Harry.

PowerPoint-ish magic

- Let’s flip Harry in what would be a truly magical and possibly somewhat painful way.
- First, swap the image’s first and last rows.
- Then, the second and second to last rows are interchanged.
- Can you visualize this?
Flipped out

We’ve swapped 0 rows.

Picture credit: http://fansshare.tumblr.com/post/16966269414/daniel-radcliffe-talks-young-harry-potter
Flipped out

We’ve swapped 100 rows.
Flipped out

We’ve swapped 200 rows.
Flipped out

We’ve swapped 300 rows.
Flipped out

We’ve swapped 400 rows.
Flipped out

We’ve swapped 450 rows.
Permutation

Each swap can be attained via matrix multiplication with a permutation matrix.

\[
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix}
\]
Diagon Alley

- We can also stroll down Diagon Alley.
- To the right, all matrix elements on or below the 1-st diagonal below the main diagonal are colored orange.
Swapping Diagon Allies

• Suppose, we keep swapping diagonals of one image for another.
• What results?
Swapping Diagon Allies

• Suppose, we keep swapping diagonals of one image for another.
• What results?
Just Kidding

Ron Weasley’s twin brothers Fred and George convinced him that the following poem was Color Change Charm:

“Sunshine, daisies, butter mellow, turn this stupid, fat rat yellow.”
Yellow belly

- The incantation of the poem was supposed to turn Ron's rat Scabbers yellow.
- All Ron got was teasing and snickers from his brothers!
Classy Charm

If linear algebra were taught in Charms Class and applied to Scabbers, how could Ron turn him yellow?

Picture credit: http://images.wikia.com/harrypotter/images/a/ab/Charms_class2.jpg
Mix up

- The color of each pixel is determined by a vector \((r, g, b)\).
- With paint, yellow is a primary color. With RGB, equal amounts of red and green make yellow.

Pixel by pixel

We use the following linear transformation on each pixel in Scabbers’ image:

$$\begin{pmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
0 & 0 & 0 \\
\end{pmatrix} \begin{pmatrix}
r \\
g \\
b \\
\end{pmatrix} = \begin{pmatrix}
\frac{1}{3} (r + g + b) \\
\frac{1}{3} (r + g + b) \\
0 \\
\end{pmatrix}$$
Dandelion-colored glasses

- This transformation produces the following change.
- The white background turned yellow which can be a bit displeasing.
Drawing the line

- Note, a white pixel (255,255,255) becomes (255,255,0), which is full yellow.
- Let’s make this region black. So, we want 255 to become 0 and take 0 to 255.
- That is, we have the points (0,255) (255,0), which becomes the line:
  \[ y = -x + 255 \]
Multicorfors

This can be done via matrices where we take
\[ N = -Y + 255(I), \]
where \( Y \) is the yellow Scabbers and \( I \) is the matrix of all ones.
Graying Mona

Suppose we take a grayscale image of the Mona Lisa.
To Ponder

How can we use linear algebra to invert the colors in this famous image where black turns white and white turns black?
Insta-math

These ideas can also create Instagram-ish effects on images.
Tools of the trade

Linear algebra isn’t quite the same as a wand from Ollivander’s Wand Shop but it does lead to some magic, at least in a Muggle-ish way.

Picture credit: http://harrypotter.wikia.com/wiki/Ollivanders_Wand_Shop
Polar makeover

- Suppose each pixel on this image is located at some coordinate in the xy-plane.
- The origin is about Marilyn’s nose and the corners are of radius 1.
Extreme Makeover

Suppose each pixel, located originally at:

\[(r, \theta)\]

is relocated to the point:

\[(r^2, \theta)\]
Extreme Makeover

Suppose each pixel, located originally at:
\[(r, \theta)\]
is relocated to the point:
\[(\sqrt{r}, \theta)\]
Photobooth math
DE in the movies

• CGI involve DEs (such as animating clothing or simulating smoke and hair on digital actors).

• More info can be found in the article *Mathematical Movie Magic* by C. and Goldman in the April 2004 issue of *Math Horizons*.
• Loci article, *Mountain of Fractals*.
• For more information, see http://mathdl.maa.org/mathDL/23/
Gallery

swap these two pieces
Gallery
http://www.neos-server.org/neos/
Gallery
Gallery
Gallery

Not red nor blue but United States
Gallery

Obomney vs Rombama

by Tim Chartier
Gallery
Gallery
Four score and seven years ago our fathers brought forth on this continent, a new nation, conceived in Liberty, and dedicated to the proposition that all men are created equal. Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battlefield of that war. We have come to dedicate a portion of that field, as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this. But, in a larger sense, we can not dedicate -- we can not consecrate -- we can not hallow -- this ground. The brave men, living and dead, who struggled here, have consecrated it far above our poor power to add or detract. The world will little note, nor long remember what we say here, but it can never forget what they did here. It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us -- that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion -- that we here highly resolve that these dead shall not have died in vain -- that this nation, under God, shall have a new birth of freedom -- and that government of the people, by the people, for the people, shall not perish from the earth.
Why? Fun!

- Computer graphics can infuse creativity into the mathematical classroom.
- That alone can be an answer to why to learn math.

*Picture credit: [Link](http://www.flickr.com/photos/26500564@N07/4146526198/in/faves-46917172@N06/)