Sections 6.3 & 6.4

Implicit Differentiation & Related Rates
**Warmup:** Find $\frac{dy}{dx}$ for each of the following implicit functions of $x$.

a) $y^3 + xy^2 = 10x$

$3y^2 \cdot \frac{dy}{dx} + x(2y \frac{dy}{dx}) + y^2 = 10$

$\frac{dy}{dx}(3y^2 + 2xy) = 10 - y^2$

$b) e^{xy} - xy = 2 - x^2$

$e^{xy}(x \frac{dy}{dx} + y) - (x \frac{dy}{dx} + y) = -2x$

$\frac{dy}{dx}[xe^{xy} - x] + ye^{xy} - y = -2x$

$$\frac{dy}{dx} = \frac{y - ye^{xy} - 2x}{xe^{xy} - x}$$

c) $x^{1/3} + y^{2/3} = 1$

$\frac{1}{3}x^{-2/3} + \frac{2}{3}y^{-1/3} \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = -\frac{1/3 x^{-2/3}}{2/3 y^{-1/3}}$$

d) $y^2 = x^2 + 1$ → $y = \pm \sqrt{x^2 + 1}$

$2y \frac{dy}{dx} = 2x$

$$\frac{dy}{dx} = \frac{x}{y} = \frac{x}{\sqrt{x^2 + 1}}$$
Strain gauge plethysmography

Want ROC in volume of limb? \( \left( \frac{dV}{dt} \right) \)

Measure ROC in circumference of limb \( \left( \frac{dC}{dt} \right) \)

\[ V = \pi r^2 L \]
\[ V = \pi \left( \frac{C}{2\pi} \right)^2 L \]
\[ \frac{dV}{dt} = 2\pi \left( \frac{C}{2\pi} \right) \left( \frac{1}{2\pi} \cdot \frac{dC}{dt} \right) \cdot L \]
\[ = \frac{C}{2\pi} \cdot L \cdot \frac{dC}{dt} \]
Tumor growth

Spherical tumor
\[ V = \frac{4}{3} \pi r^3 \]

Suppose \( r \) expanding at constant rate \( K \)

What is rate of change in volume?

\[ V \text{ is a fcn of } t \]

\[ r \text{ is a fcn of } t \]

\[ \frac{dV}{dt} = 4 \pi r^2 \frac{dr}{dt} = 4 \pi r^2 \cdot K \quad \text{(proportional to S.A.)} \]

\[ A = \pi r^2 \quad C = 2\pi r \]
What is ROC in \( l \) as spider runs along the floor, away from wall

\[ l = \text{fun of } t \quad \frac{dl}{dt} \quad \text{rate of silk production} \]

\[ h = \text{constant} \]

\[ x = \text{fun of } t \quad \frac{dx}{dt} \quad \text{speed of spider} \]

\[ x^2 + h^2 = l^2 \]

\[ 2x \frac{dx}{dt} + 0 = 2l \frac{dl}{dt} \]

\[ \frac{dl}{dt} = \frac{x}{l} \frac{dx}{dt} \quad \text{constant} \]

Increasing with time

(as spider runs from wall at a constant rate, silk comes out faster + faster)
IV drip

\( \frac{dV}{dt} \) should be constant \((2 \text{ cc/min})\)

How much faster is level dropping when halfway down trough section than when in cylinder section?

In cylinder: \( V = \pi r^2 h \) \( \pi = \text{constant} = 6 \)

\[ \frac{dV}{dt} = \pi r^2 \frac{dh}{dt} \quad \frac{dh}{dt} = \frac{\frac{dV}{dt}}{\pi r^2} \]

\[ \approx -\frac{2}{36\pi} \]

\( \approx -0.017 \)

In trough, \( V = \frac{1}{2} h \cdot b \cdot l \) \( h \) is far of \( t \)

\( b = \frac{3}{2} h \)

\[ V = \frac{1}{2} h (\frac{3}{2} h) l = \frac{3}{4} h^2 l = \frac{3}{4} h^2 (12) = 9\ h^2 \]

\[ \frac{dV}{dt} = 18h \frac{dh}{dt} \Rightarrow \frac{dh}{dt} \approx 0.03 \]