Section 2.2
Logarithmic Functions
Defn of logarithm (in your own words)

- Logs are used to solve exponential equations (use them whenever the variable is in the exponent)

- Log is the number to which the base is raised to get a given number (i.e., \( \log = \text{power} \))

- Log is inverse of exponential function
Calculating logs

\( \log_a x = y \)

\( a^y = x \)

\( \ln a^y = \ln x \)

\( y \ln a = \ln x \)

\( y = \frac{\ln x}{\ln a} \)

or

\( y = \frac{\log x}{\log a} \)
Solve using logs

a) \(2^x = 16\)
   \(\log_2 2^x = \log_2 16\)
   \(x = \log_2 16\)
   \(10^{\log_2 16} = \log_10 16\)
   \(x = \frac{\log_{10} 16}{\log_{10} 2}\)

b) \(2^x = 24\)
   \(x = \log_2 24 = \frac{\ln 24}{\ln 2}\)

c) \(e^x = 12\)
   \(x = \ln 12\)

d) \(10^{x^2} = 42\)
   \(\log_{10} 10^{x^2} = \log_{10} 42\)
   \(x^2 = \log_{10} 42\)
   \(x = \pm \sqrt{\log_{10} 42} \approx \pm 1.27\)

e) \(\ln(x) + \ln(x^2) = -2\)
   \(\ln(x \cdot x^2) = -2\)
   \(3 \ln x = -2\)
   \(\ln x = -\frac{2}{3}\)
   \(x = e^{-\frac{2}{3}}\)

f) \(\log(x-2) - \log(x) = 6\)
   \(\log \frac{x-2}{x} = 6\)
   \(10^6 \cdot \frac{x-2}{x} = 10\)
   \(x - 2 = x \cdot 10^6\)
   \(x = \frac{2}{999,999}\)
   \(x - 2 = 2 \times 10^6\)
RQ #3

Start w/ 10 cells, double every 30 minutes.
Write number of cells at time $t$ (in hours) in the form

$$P(t) = P_0 e^{rt}$$

$P_0 = 10$

$r = \ln 4$

$$P(t) = 10 \cdot e^{\ln 4 \cdot t} \approx 10 \cdot e^{1.39 \cdot t}$$
**Doubling Time**

\[ P(t) = P_0 \cdot a^t \]

\[ 2P_0 = P_0 \cdot a^t \]
\[ a^t = 2 \]
\[ t = \frac{\ln 2}{\ln a} \]

\[ P(t) = P_0 \cdot e^{rt} \]

\[ 2P_0 = P_0 \cdot e^{rt} \]
\[ e^{rt} = 2 \]
\[ rt = \ln 2 \]
\[ t = \frac{\ln 2}{r} \]

[what does “a” represent in terms of growth?]

\[ P_0 \cdot 2^t = P_0 \cdot (1 + r)^t \]
\[ \Rightarrow a = 1 + r, \text{ where } r \text{ is the } \%	ext{ change per unit of time} \]

[what does “r” represent in terms of growth?]
\[ t = \log_a \left( \frac{1}{2} \right) = (\log_a(1)) - (\log_a(2)) \]
\[ = - \log_a 2 \]
\[ = - \frac{\ln 2}{\ln a} \]