Section 8.3
Volume & Average Value
PUMPKINS!!
(De) Constructing a solid of revolution
From this picture (without peeking), derive the formula for the volume of a solid of revolution.

\[ V = \lim_{\Delta x \to 0} \sum_{i=1}^{n} \pi (f(x_i))^2 \Delta x = \int_{a}^{b} \pi f(x)^2 \, dx \]
Find the volume of the solid of revolution formed by rotating about the x-axis the region bounded by 

\[ y = 4 - x^2, \quad x = -1, \quad x = 1 \]

\[ V = 2 \pi \int_{-1}^{1} (4 - x^2)^2 \, dx \]

\[ = \pi \left[ 16 - 8x^2 + x^4 \right]_{-1}^{1} \]

\[ = \pi \left( 16 - \frac{8}{3} + \frac{1}{5} \right) \]

\[ = \pi \left( 16 \right) \]
Use a solid of revolution to determine the volume of a right circular cone with radius $r$ and height $h$.

\[ y = \frac{r}{h} x \]

\[ \int_0^h \pi \left( \frac{r}{h} x \right)^2 \, dx \]

\[ = \pi \int_0^h \frac{r^2}{h^2} x^2 \, dx \]

\[ = \frac{\pi r^2}{h^2} \left[ \frac{x^3}{3} \right]_0^h \]

\[ = \frac{\pi r^2}{h^2} \frac{h^3}{3} - 0 = \frac{\pi r^2 h}{3} \]
Average value of a function over interval \([a, b]\) = height of rectangle (with width \(b-a\)) that has same area as the area under \(f(x)\) b/w \(x=a\) and \(x=b\).

Area of rectangle = \((b-a) \cdot h\)

Area under \(f(x)\) = \(\int_{a}^{b} f(x)\,dx\)

\((b-a) \cdot h = \int_{a}^{b} f(x)\,dx\)

\(h = \frac{1}{b-a} \int_{a}^{b} f(x)\,dx\)