Section 7.3
Area & the Definite Integral
Approximate the area under the graph of \( f(x) = \frac{1}{x} \) from \( x = 1 \) to \( x = 3 \) (above the x-axis)

\( n = 4 \)

a) left endpoints

b) right endpoints

c) avg of (a) and (b)

d) midpoints

\[
a) \quad f(1) \times 0.5 + f(1.5) \times 0.5 + f(2) \times 0.5 + f(2.5) \times 0.5 \\
= \frac{1}{1} \times 0.5 + \frac{1}{1.5} \times 0.5 + \frac{1}{2} \times 0.5 + \frac{1}{2.5} \times 0.5 \approx 1.28
\]

\[
b) \quad f(1.5) \times 0.5 + f(2) \times 0.5 + f(2.5) \times 0.5 + f(3) \times 0.5 \approx 0.95
\]
When do we know we get an overestimate w/ left endpoints?

Whenever $f(x)$ is decreasing
Is midpoint always better estimate than left (or right)?
Application: IV drips at rate \(6-x\) ml/hr for 6 hours.

a) Approximately how much fluid is delivered in this time?
b) Write the quantity as a definite integral

\[ \int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \, \Delta x \]

c) Find the exact quantity using geometry